

Transformations Review

①

$$y = f(x)$$

$$(x, y)$$

$$(2, 3)$$



$$y = 3f\left(\frac{1}{4}x\right)$$

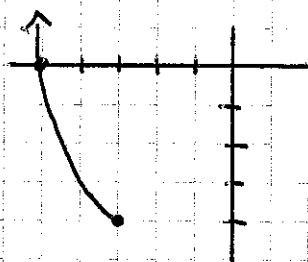
$$(4x, 3y)$$

$$(8, 9)$$

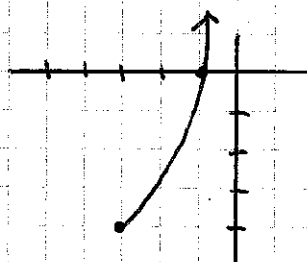
②

Divide the graph in half at its axis of symmetry. $x = -3$

Option 1 $x \leq -3$



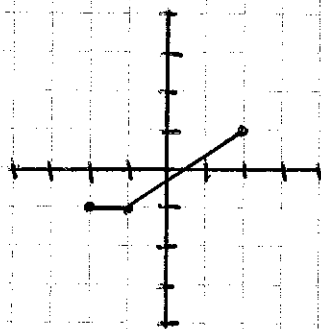
Option 2 $x \geq -3$



③

$y = f(-x)$ is a reflection in the y-axis so for every point (x, y) on $f(x)$ there will be a point $(-x, y)$ on $f(-x)$. Since the y-coord. is unaffected, the range is unaffected.

④



reflect in x-axis

⑤

$$y = \sqrt{2x-2}$$

$$y = \sqrt{2(x-1)}$$

is $y = \sqrt{x}$ H.C. by $\frac{1}{2}$ and right 1

$$y = \sqrt{x}$$

$$(0, 0)$$

$$(1, 1)$$

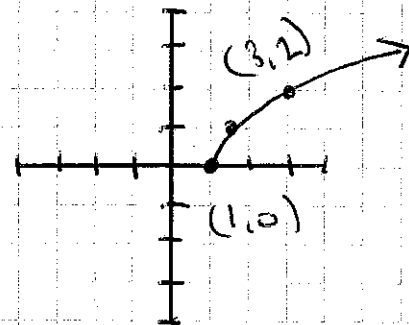
$$(4, 2)$$

$$y = \sqrt{2(x-1)}$$

$$(1, 0)$$

$$(1.5, 1)$$

$$(3, 2)$$



⑥

$$y = 2x - 6$$

$$x = 2y - 6$$

$$2y = x + 6$$

$$y = \frac{x}{2} + 3$$

switch x and y

isolate y

⑦

$y = f(x-3) + 5$ is $y = f(x)$ translated right 3 and up 5.

⑧

$y = f(2x-6)$ or $y = f(2(x-3))$ is the graph of $y = f(x)$ horizontally compressed by $\frac{1}{2}$ then translated right 3.

⑨

reflecting in $y = x$ means the x and y coord of each point switch.

$$\therefore F^{-1}(x) = \{(4, -3), (7, 2), (6, 8)\}$$

$$D: \{4, 7, 6\}$$

(10)

$$y = F(x)$$

$$(x, y)$$

$$(3, -2)$$



$$y = 2F(x+1)$$

$$(x-1, 2y)$$

$$(2, -4)$$

(11)

$$y = 4 - x$$

$$x = 4 - y$$

$$x - 4 = -y$$

$$y = -(x - 4)$$

$$y = 4 - x$$

so $F^{-1}(x) = 4 - x$

∴ $F^{-1}(x) = F(x)$

(12)

$y = \sqrt{x+1} - 2$ is
and down 2

$y = \sqrt{x}$ translated left 1

$$y = \sqrt{x}$$

$$(0, 0)$$

$$(1, 1)$$

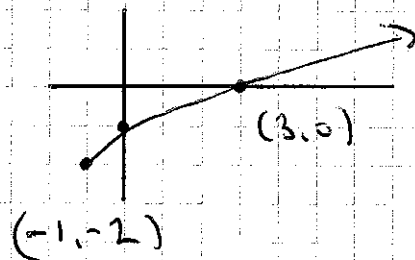
$$(4, 2)$$

$$y = \sqrt{x+1} - 2$$

$$(-1, -2)$$

$$(0, -1)$$

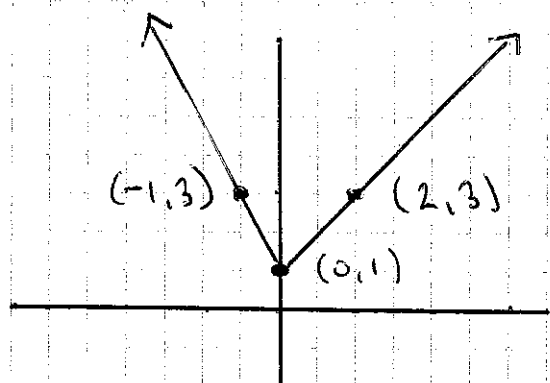
$$(3, 0)$$



(13)

first multiply the y -values by 2, then
move the graph up 5

14



$$g(x) = f(x-2) + 3$$

right 2

up 3

15

$2f(2x)$ means $(x, y) \rightarrow (\frac{1}{2}x, 2y)$

$$\therefore (4, -3) \rightarrow \boxed{(2, -6)}$$

16

since x and y are switched, this is an inverse relation, or a reflection in the line $y = x$.

17

$$y = (x+1)^2 \quad x \leq -1$$

$$x = (y+1)^2 \quad \text{switch } x \text{ and } y$$

$$\pm \sqrt{x} = y+1 \quad \text{isolate } y$$

$$y = \pm \sqrt{x} - 1$$

$$y \leq -1$$

(Domain of f becomes range of f^{-1})

$$\therefore y = -\sqrt{x} - 1$$

$$f^{-1}(x) = -\sqrt{x} - 1$$

18

Split the graph at its axis of symmetry $x = 1$.

Option 1 $x \leq 1$

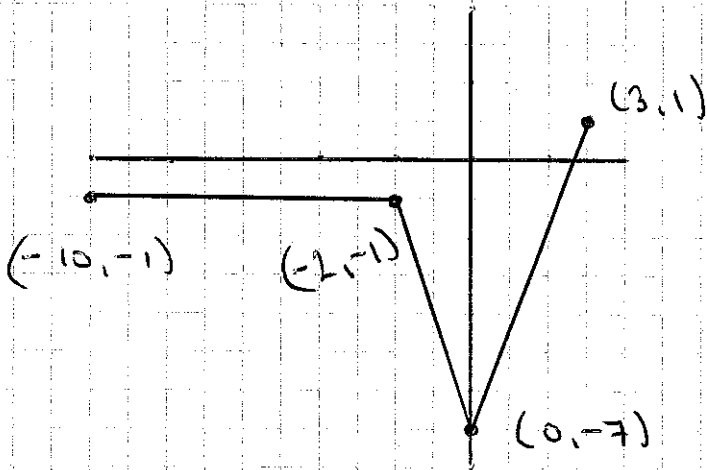
Option 2 $x \geq 1$

(19) reflection in x , so $y = -f(x)$

$$y = -f(x)$$
$$= -(x^2 + 4)$$

$$y = -x^2 - 4$$

(20) v.s. by 2 then down 3



(21) 3 units right $\rightarrow y = f(x-3)$

$$\therefore y = x^2 - x + 2$$
$$= (x-3)^2 - (x-3) + 2$$
$$= x^2 - 6x + 9 - x + 3 + 2$$

$$y = x^2 - 7x + 14$$

(22) $y = \sqrt{x}$ has a domain $x \geq 0$

$y = \sqrt{-4x}$ is a H.C. by $-\frac{1}{4}$, so multiply x -coord by $-\frac{1}{4}$. Do the same to the domain.

So $y = \sqrt{-4x}$ has domain $x \geq 0$

(23)

$$f(x)$$

$$(x, y)$$

$$(-3, 5)$$



$$f(-x)$$

$$(-x, y)$$

$$\boxed{(3, 5)}$$

(24)

$y = -\sqrt{3(x+1)}$ is $y = \sqrt{x}$ H.C. by $\frac{1}{3}$,
reflected in x -axis, then moved left 1

$$y = \sqrt{x}$$

$$(x, y)$$

$$(0, 0)$$

$$(4, 2)$$

$$(9, 3)$$

$$y = -\sqrt{3(x+1)}$$

$$\left(\frac{1}{3}x - 1, -y\right)$$

$$(-1, 0)$$

$$\left(\frac{1}{3}, -2\right)$$

$$(2, -3)$$

